On the Power Efficiency of Sensory and Ad-Hoc Wireless Networks

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Wireless Networks

- wireless is a *shared* medium
  - network is a fully-connected graph
  - bandwidth is precious
- distinguishing features
  - interference
  - path-loss
  - fading

Traditional methods attempt to combat these “deficiencies”.

A better approach is to *exploit* them.

Sensory Networks and Ad-Hoc Networks

Sensory Networks:

- applications: environmental, surveillance, etc.
- all information intended for a single receiver
- sensors may only need to occasionally transmit information

Ad-Hoc Networks:

- no infrastructure
- users "somehow" cooperate in a distributed fashion to communicate
- can have many users acting simultaneously as transmitters or receivers

Some Scaling Laws

How does the capacity of such networks scale in n, the number of nodes?

- for sensory networks capacity scales as $O(\log n)$ (Gastpar and Vetterli, 2002)
- for ad-hoc networks capacity scales as $O(\sqrt{n})$ (Gupta and Kumar, 2000)

Both are overall discouraging results:

- in sensory networks, the capacity per participating node is $O(\frac{\log n}{n})$
- in ad-hoc networks, the capacity per participating node is $O(\frac{1}{\sqrt{n}})$

In either case, these are diminishing returns for the nodes' energy investment and participation in the communications

Power Efficiency

- in wireless networks, especially sensory networks, power consumption is a major bottleneck
- this had led many researchers to study bits/energy

Following Verdu (2002), we shall rather look at (bits/sec)/(energy/sec), i.e., at *power efficiency* defined as

$$\eta = \frac{\text{capacity}}{\text{transmit power}}$$

For an AWGN channel this is

$$\eta = \frac{\log(1 + \sigma_s^2/\sigma_n^2)}{\sigma_s^2}$$

Power Efficiency

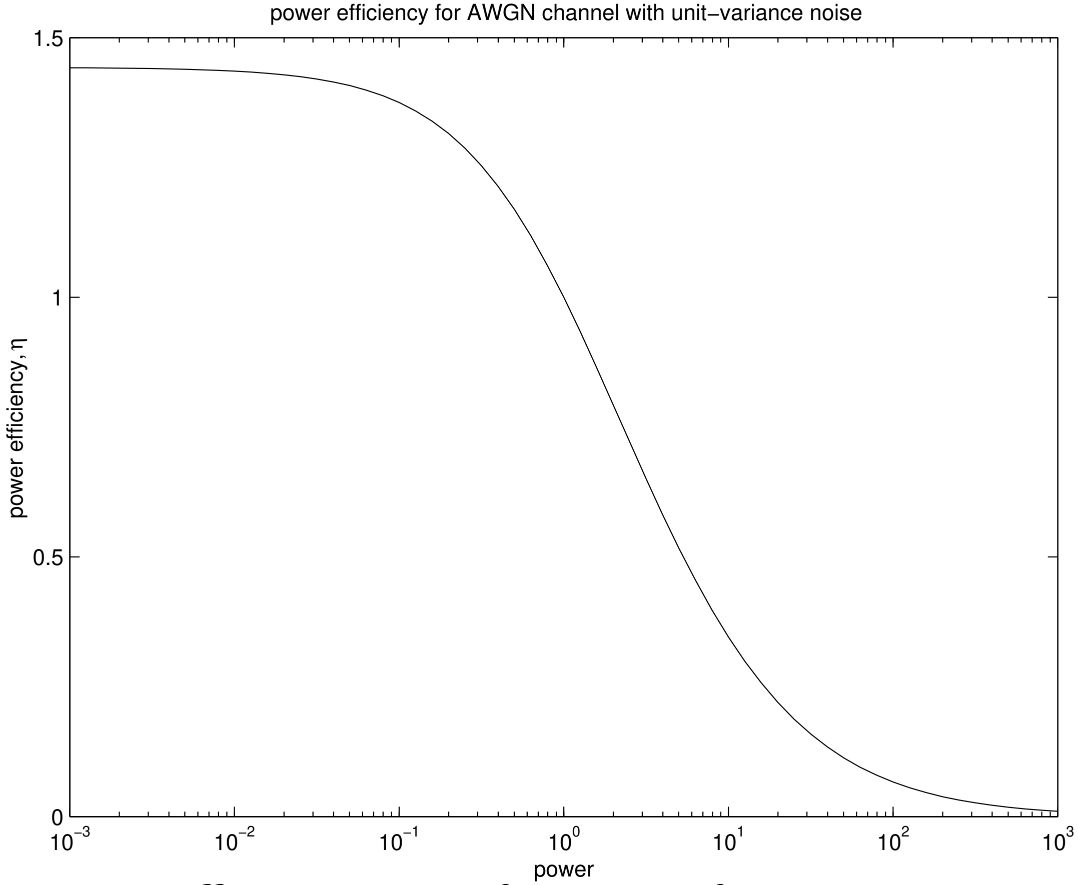


Figure 1: Power efficiency as a function of transmit power for an AWGN channel with unit-variance noise

Clearly,

- as $\sigma_s^2 \to \infty$:

$$\eta \to \frac{\log \sigma_s^2}{\sigma_s^2} \to 0,$$

  which means we are power in-efficient at high SNR

- as $\sigma_s^2 \to 0$:

$$\eta = \frac{\log(1 + \sigma_s^2/\sigma_n^2)}{\sigma_s^2} \to \frac{\log e}{\sigma_n^2} = O(1),$$

  which means we are power efficient at low SNR

This was all from a single-user perspective.

What happens in a network setting?...

Multi-Antenna Power Efficiency

To gain some insight, consider an n-transmit single-receive multi-antenna channel, with channel matrix:

$$H = \left[\begin{array}{cccc} h_1 & h_2 & \dots & h_n \end{array} \right] \quad , \quad E|h_i|^2 = 1$$

- if the channel matrix is *known* to the transmitter (beamforming):

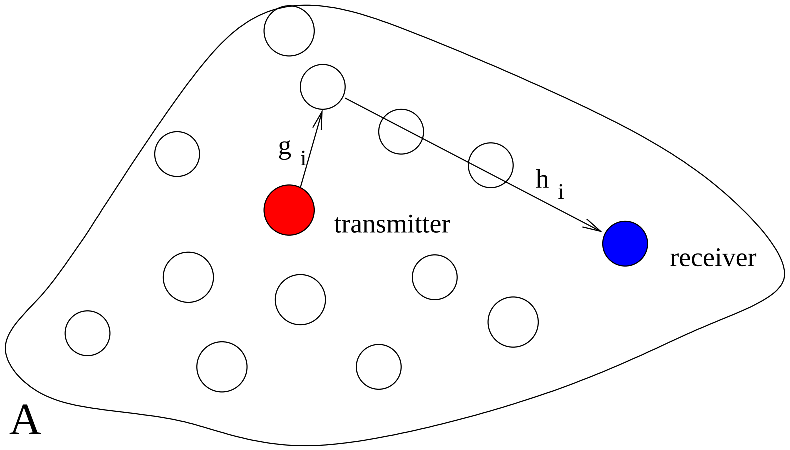
$$\eta = \frac{E \log \left(1 + \frac{\sigma_s^2}{\sigma_n^2} (\sum_{i=1}^n |h_i|^2)^2 \right)}{n \sigma_s^2} \to \frac{n \log e}{\sigma_n^2} = O(n)$$

- if the channel matrix *unknown* to the transmitter (Foschini-Telatar):

$$\eta = \frac{E \log \left(1 + \frac{\sigma_s^2}{\sigma_n^2} \sum_{i=1}^n |h_i|^2 \right)}{n \sigma_s^2} \to \frac{\log e}{\sigma_n^2} = O(1)$$

From a power efficiency point of view, it only pays off to have multiple transmit antennas if the channel is known at the transmitter.

Sensory Networks

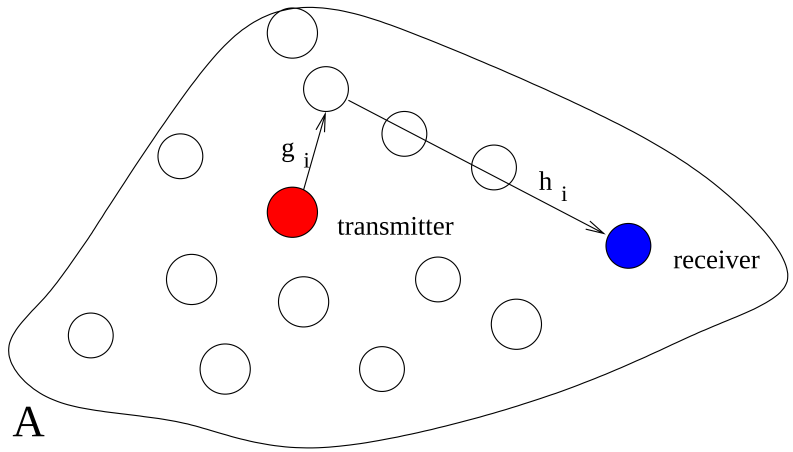


Consider a fixed area A and randomly choose n nodes in this area and a transmit receive pair.

- gains from transmitter to each node: $g_i, \quad i = 1, \ldots, n$
- gains from each node to receiver: $h_i, \quad i = 1, \ldots, n$

We will be interested in the asymptotics when A is fixed and $n \to \infty$.

Sensory Networks



- **Assumptions:**
  - $\{g_i, h_i\}$ are subject to independent fading. Averaged over the fading and the point placement of the nodes, they are unit-variance with finite fourth-order moment
  - system is synchronous
  - each node i knows its local connections, g_i and h_i, but not the rest of the network

A "Listen and Transmit" Protocol

Communication is divided into two intervals:

1. The *listen* interval: the transmitter transmits the signal s, with power p, and all other nodes are silent, but listen:

$$q_i = g_i s + v_i, \quad i = 1, \ldots, n$$

2. The *transmit* interval: each node transmits with power σ_s^2 the signal

$$t_i = \frac{\sigma_s}{\sqrt{\sigma_n^2 + p}} g_i^* h_i^* q_i \quad i = 1, \ldots n$$

The received signal is

$$\begin{aligned} r = \sum_{i=1}^{n} h_i t_i + w &= \sum_{i=1}^{n} |h_i|^2 \frac{\sigma_s}{\sqrt{\sigma_n^2 + p}} g_i^* q_i + w \\ &= \sum_{i=1}^{n} \frac{\sigma_s}{\sqrt{\sigma_n^2 + p}} \left(|h_i g_i|^2 s + |h_i|^2 g_i^* v_i \right) + w \end{aligned}$$

A Digression: Does this Achieve Capacity?

Note that what we are doing is essentially *distributed beamforming.*

Thus the signal adds up coherently and the noise non-coherently:

$$\mathrm{SNR} = \frac{\frac{n^2\sigma_s^2}{\sigma_n^2+p}}{\sigma_n^2(1+\frac{n\sigma_s^2}{\sigma_n^2+p})} = \frac{n^2\sigma_s^2 p}{\sigma_n^2(\sigma_n^2+p+n\sigma_s^2)}$$

As $n \to \infty$,

$$\mathrm{SNR} \approx n\frac{p}{\sigma_n^2},$$

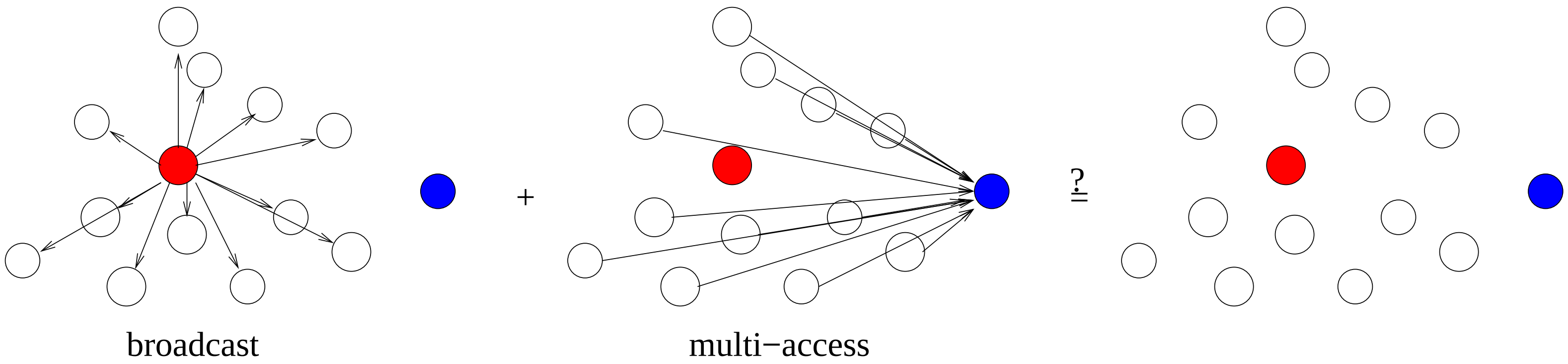
and so

$$C_{listen-transmit} \;\geq\; \frac{1}{2}\log n + O(1)$$

Therefore the "listen and transmit" protocol achieves $O(\log n)$, which is the same order as capacity.

Is the Total Larger than the Sum of the Parts?

A common approach in many wireless networks is to use reliable communications between nodes combined with routing.



In this approach

$$C = \min(C_{broadcast}, C_{multi-access}).$$

But,

$$C_{broadcast} = O(\log\log n) \quad , \quad C_{multi-access} = O(\log n)$$

which leads to $C = O(\log\log n)$ and is well short of $O(\log n)$.

Back to Power Efficiency

$$\mathrm{SNR} = \frac{n^2 \sigma_s^2 p}{\sigma_n^2(\sigma_n^2 + p + n\sigma_s^2)}$$

Assume $p = O(n^{-\epsilon})$ and $n\sigma_s^2 = O(n^{-\epsilon})$ for some $\epsilon > 0$. Then

$$\mathrm{SNR} = \frac{n(n\sigma_s^2)p}{\sigma_n^4} = O(n^{1-2\epsilon}) \quad \text{and} \quad \eta = O(\frac{n^{1-2\epsilon}}{n^{-\epsilon}}) = O(n^{1-\epsilon})$$

Therefore $\epsilon > 1/2$ and so the best we can do is:

$$\eta = O(\sqrt{n})$$

Precise Statement

The above argument was very rough. Here is the precise statement.

Theorem 1 *Consider a fixed area A and randomly select n nodes and a transmit/receive pair in this area. Then*

$$Prob\left(\eta > k_1\sqrt{n}\right) > 1 - \frac{k_2}{n},$$

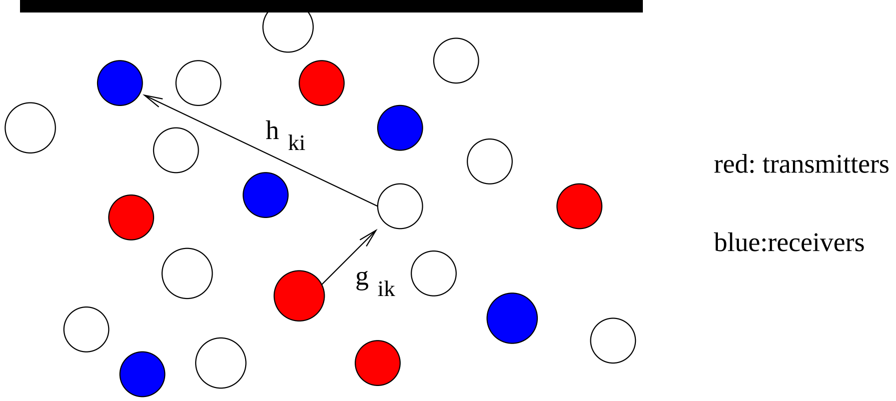
where k_1 and k_2 are positive constants independent of n, but that depend on A and the fading statistics.

Power Efficiency of Sensory Networks

- the power efficiency is thus $O(\sqrt{n})$
- compared to a single link, for a fixed transmission rate, the total power consumption in the network can be reduced by a factor of $\frac{1}{\sqrt{n}}$
- this is even better than the $n \times 1$ multi-antenna channel with no channel state information ($O(1)$), though not as good as the $n \times 1$ case with channel state information ($O(n)$)
- the protocol is double-hop and exploits, rather than avoids, interference (it essentially does distributed beam-forming)
- there is built-in fairness: the transmitter transmits with power $p = O(\frac{1}{\sqrt{n}})$ and all other nodes with power $\sigma_s^2 = O(\frac{1}{n\sqrt{n}})$

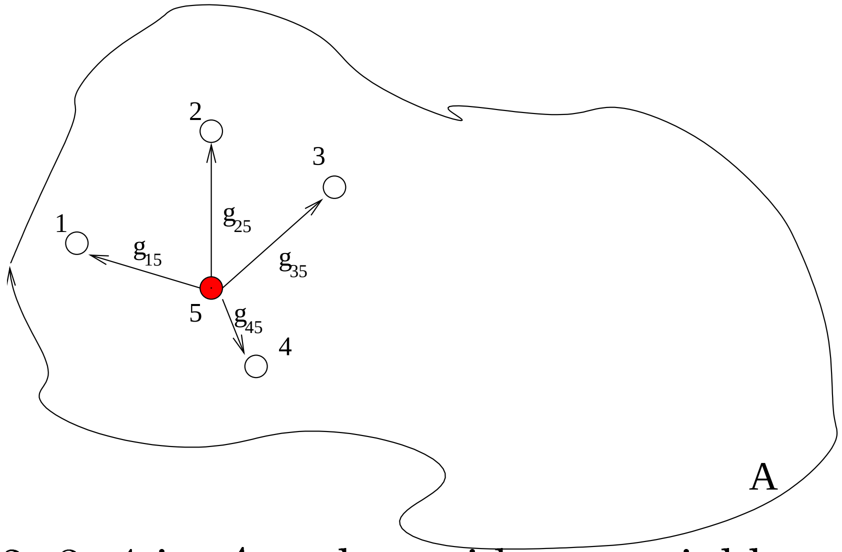
Thus, from a power efficiency point of view, it pays off to network!

Ad-Hoc Networks



- Fixed area A, n nodes, $r \leq \sqrt{n}$ users transmitting, r users receiving
- gains from transmitters to each node: $g_{ik}, \quad i = 1, \ldots, r \quad k = 1, \ldots, n$
- gains from each node to receivers: $h_{ki}, \quad k = 1, \ldots, n \quad i = 1, \ldots, r$
- **Assumptions:**
  - $\{g_{ik}, h_{ki}\}$ subject to independent fading. Averaged over fading and point placement, unit-variance, finite fourth-order moment
  - system synchronous
  - each node k knows its local connections, g_{ik} and h_{kj}, but not the rest of the network

Technical Assumption



Fix four points 1, 2, 3, 4 in A and consider a variable point 5. Then averaged over the fading and the point placement of 5 we assume:

$$Eg_{15}g_{25}g_{35}g_{45} = 0.$$

This is true if the g_{ij} are zero-mean when averaged over the fading:

$$Eg_{15}g_{25}g_{35}g_{45} = E_5\left[E_{|5}g_{15}g_{25}g_{35}g_{45}\right] = E_5\,\underbrace{E_{|5}g_{15}}_{0}\,\underbrace{E_{|5}g_{25}}_{0}\,\underbrace{E_{|5}g_{35}}_{0}\,\underbrace{E_{|5}g_{45}}_{0} = 0.$$

Back to "Listen/Transmit" Protocol

We now have interference. Let's look at our previous protocol.

1. The listen interval: Each of the r transmit users transmits the signal s_i with power p. All other nodes are silent and measure the signals

$$q_k = \sum_{i=1}^{r} s_i g_{ik} + v_k, \quad i = 1, \ldots, r \quad k = 1, \ldots, n$$

2. The transmit interval: Each of the nodes now has to transmit a signal. Let us assume that all it can transmit is a scaled version of what it has previously received:

$$t_k = d_k q_k, \quad k = 1, \ldots, n$$

such that the transmit power is σ_s^2. But how to choose d_k?

Defining the transmit vector $s = \begin{bmatrix} s_1 & s_2 & \dots & s_r \end{bmatrix}$, the receive vector $y = \begin{bmatrix} y_1 & y_2 & \dots & y_r \end{bmatrix}$ and the matrices

$$G = \begin{bmatrix} g_{11} & g_{12} & \dots & g_{1n} \\ g_{21} & g_{22} & \dots & g_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ g_{r1} & g_{r2} & \dots & g_{rn} \end{bmatrix} \quad , \quad H = \begin{bmatrix} h_{11} & h_{12} & \dots & h_{1r} \\ h_{21} & h_{22} & \dots & h_{2r} \\ \vdots & \vdots & \ddots & \vdots \\ h_{n1} & h_{n2} & \dots & h_{nr} \end{bmatrix}$$

we may write

$$y = s \cdot G \cdot \mathrm{diag}(d_1, \dots, d_n) \cdot H$$

Note that if $n \geq r^2$, the d_i can be chosen such that

$$G \cdot \mathrm{diag}(d_1, \dots, d_n) \cdot H = I_r, \tag{1}$$

which means that the channel is diagonalized and the interference suppressed! (We thus achieve the $O(\sqrt{n})$ capacity.)

But this can only be done if each node knows all the network gains so that it can solve (1). This is not allowed.

What to do?

- Let each node estimate each of the r transmitted signals:

  $$\hat{s}_i = g_{ik}^* q_k, \quad i = 1, \ldots, r \quad k = 1, \ldots, n$$

  Of course, this will be a lousy estimate since the SINR (signal-to-interference-ratio) is $\frac{p}{\sigma_n^2 + (r-1)p}$.

- Let each node attempt to "coherently add" these estimates:

  $$t_k = \frac{1}{\sqrt{r}} \cdot \frac{\sigma_s}{\sqrt{\sigma_n^2 + rp}} \sum_{i=1}^{r} h_{ki}^* \hat{s}_i = \frac{\sigma_s}{\sqrt{r(\sigma_n^2 + rp)}} \sum_{i=1}^{r} h_{ki}^* g_{ik}^* q_k$$

  Note therefore that

  $$d_k = \frac{\sigma_s}{\sqrt{r(\sigma_n^2 + rp)}} \sum_{i=1}^{r} h_{ki}^* g_{ik}^*$$

  depends only on *local* knowledge of the network gains.

The received signal at the j-th receiver is:

$$y_j = \sum_{k=1}^{n} t_k h_{kj} + w_j.$$

Close inspection of the received signal reveals that it consists of $n+1$ noise terms (one from each of the n nodes) and nr^2 signal terms, of which

- there are n terms of the desired signal s_j that add up *coherently* (one from each of the n nodes)
- there are $n(r-1)$ terms of the desired signal s_j that add up *non-coherently* ($r-1$ from each of the n nodes)
- there are $nr(r-1)$ interference terms (r from each interferer and each node)

Therefore

$$\mathrm{SINR} = \frac{\frac{(n^2+n(r-1))\sigma_s^2 p}{r(\sigma_n^2+rp)}}{\frac{nr(r-1)\sigma_s^2 p}{r(\sigma_n^2+rp)} + \frac{n\sigma_n^2\sigma_s^2}{r(\sigma_n^2+rp)} + \sigma_n^2} \approx \frac{rpn^2\sigma_r^2}{r^2(\sigma_n^2+rp)(\sigma_n^2+n\sigma_r^2)}$$

A Digression: Does this Achieve Capacity?

Note that with our choice of d_k, the SIR has been boosted from $\frac{1}{r}$ to $\frac{n}{r^2} = \frac{1}{r} \cdot \frac{n}{r}$.

Thus,

$$C_{ah} \;\geq\; \frac{r}{2} \log\left(1 + \frac{n}{r^2}\right).$$

The above bound is maximized with the choice $r = \sqrt{n}$, which yields

$$C_{ah} \;\geq\; \frac{\sqrt{n}}{2}.$$

We therefore have the bound

$$C_{ah} \;\geq\; O(\sqrt{n}),$$

which is the Gupta-Kumar result.

Power Efficiency of Ad-Hoc Networks

Assuming the technical condition:

Theorem 2 *Consider a fixed area A and randomly select n nodes and $r \leq O(n^{\frac{1-\epsilon}{2}})$ transmit/receive pairs in this area. Then*

$$Prob\left(\eta > k_1\sqrt{n}\right) > 1 - \frac{k_2}{n^\epsilon},$$

where k_1 and k_2 are positive constants independent of n, but that depend on A and the fading statistics.

Without the technical condition:

Theorem 3 *Consider a fixed area A and randomly select n nodes and $r \leq O(n^{\frac{1-\epsilon}{3}})$ transmit/receive pairs in this area. Then*

$$Prob\left(\eta > k_1\sqrt{n}\right) > 1 - \frac{k_2}{n^\epsilon},$$

where k_1 and k_2 are positive constants independent of n, but that depend on A and the fading statistics.

Power Efficiency of Ad-Hoc Networks

- *provided* $r \leq O(\sqrt{n})$, the power efficiency is $O(\sqrt{n})$
- if we fix the rate for each of the r transmit/receive pairs, then the total power consumption of the network reduces by a factor of $\frac{1}{\sqrt{n}}$.
- this is as good as a sensory network
- rather than diminishing rewards, we are reaping benefits from the increase in the size of the network
- there is built-in fairness: the r transmitters transmit with power $p = O(\frac{1}{\sqrt{n}})$ and all other nodes with power $\sigma_s^2 = O(\frac{r}{n} \cdot \frac{1}{\sqrt{n}})$
- again, the protocol is double-hop and exploits, rather than avoids, interference

Final Remarks and Further Questions

- if one focuses on power efficiency it apparently does pay off to consider sensory and wireless ad-hoc networks
- the net benefit is larger than the sum of the individual benefits
- to get encouraging asymptotics, rather than attempting to see how the rate scales, the key is to fix the rate and study how the power can be lowered
- interference is, in fact, useful
- **some questions:**
  - are our bounds tight?
  - what are the bandwidth power trade-offs? (this requires studying the second-derivative of capacity at zero SNR)