

## Estimation-Based Multi-Channel Adaptive Algorithm for Filtered-LMS Problems<sup>1</sup>

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### Abstract

This paper presents an estimation-based adaptive filtering algorithm for the multi-channel Filtered-LMS problems where a number of adaptively controlled secondary sources use multiple reference signals to cancel the effect of a number of primary sources (*i.e.* disturbance sources) as seen by a number of error sensors. We show that our estimation based approach easily extends to the multi-channel case, and that it maintains all of the stability and performance features of the single-channel solution.

The problem of noise cancellation in a one dimensional acoustic duct, and a structural vibration control problem are chosen to examine the main characteristics of the proposed multi-channel adaptive algorithm. The performance of the new multi-channel adaptive algorithm is compared to the performance of a multi-channel implementation of the FxLMS algorithm in these cases, and it is shown that the new algorithm provides a faster response, with improved transient behavior and steady-state performance.

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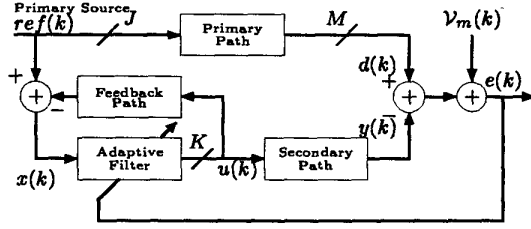
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### 1 Background

For a wide variety of applications, such as equalization in wireless communication when more than one receiver/transmitter are involved [1, 2], or active control of sound and vibration in cases where the acoustic environment or the dynamic system of interest is complex and a number of *primary* sources excite the system [3], multi-channel adaptive filtering (control) algorithms are needed. Compared to single-channel algorithms however, multi-channel adaptive schemes are significantly more complex [4]. As Reference [4] points out, successful application of the classical multi-channel adaptive algorithms has been limited to cases involving repetitive noise with a few harmonics [5]. This observation agrees with the results in Ref. [6] where a significant noise reduction is only achieved for periodic disturbances<sup>1</sup>. This paper presents a new systematic approach to the multi-channel adaptive filtering/control synthesis for the filtered-LMS problems. This new approach is based on an estimation-interpretation of the adaptive filtering/control problem which has been used to develop the Estimation-Based Adaptive Filtering (EBAF) algorithm for both FIR and IIR adaptive filters in single channel Filtered-LMS problems and is experimentally tested for the noise cancellation in a one-dimensional acoustic duct [7]. In contrast to the classical ap-

<sup>1</sup>Another interesting conclusion in Ref. [6] is the following: the performance of a Multi-Channel implementation of the FxLMS algorithm is similar to the performance of an  $H_\infty$  controller which was directly designed for noise cancellation.



**Fig. 1:** General block diagram for a multi-channel Active Noise Cancellation (ANC) problem

proaches to the multi-channel adaptive filter/control design, we show that the multi-channel implementation of the new estimation-based approach is virtually identical to the single channel case. Furthermore, the analysis of the multi-channel adaptive system in the new framework is shown to be a straightforward extension of the analysis used in the single channel case.

## 2 Estimation-Based Adaptive Algorithm for Multi Channel Case

Figure 1 shows the general block diagram of a multi-channel ANC system in which reference signals can be affected by the output of the adaptive filters. Simulations in this chapter, however, are based on the assumption that the effects of feedback are negligible and that the reference signal is available to the adaptation scheme through a noisy measurement. Measurement noise is independent from the reference signal. In Figure 1, the ANC adaptive filter has  $J$  reference input signals denoted by  $\mathbf{x}^T(k) = [x_1(k) \ x_2(k) \ \cdots \ x_J(k)]$ . The controller generates  $K$  secondary signals that are elements of the control vector  $\mathbf{u}^T(k) = [u_1(k) \ u_2(k) \ \cdots \ u_K(k)]$ . Therefore, a  $K \times J$  matrix of adaptive FIR filters can be used to describe the adaptive control block in Figure 1. Defining  $L$  as the length of each adaptive FIR filter, the reference signal vector is

$$\mathbf{X}^T(k) = [\mathbf{x}_1^T(k) \ \mathbf{x}_2^T(k) \ \cdots \ \mathbf{x}_J^T(k)] \quad (1)$$

where  $\mathbf{x}_j^T(k) = [x_j(k) \ x_j(n-1) \ \cdots \ x_j(n-L+1)]$  is the last  $L$  samples of the  $j$ -th reference signal. For a description of the multi-channel implementation of the FxLMS adaptive algorithm, see [4], Chapter 5, and the references therein.

The underlying concept for the estimation based adaptive filtering algorithm is the same for both single-channel and multi-channel systems. Therefore, an estimation interpretation identical to the one in the single-channel case can be used to translate a given adaptive filtering (control) problem into an equivalent robust estimation problem. See [7] for a detailed treatment of the steps involved in this translation.

As in the single-channel case, state space models for the adaptive filter and the secondary path are used to construct an approximate model for the unknown primary path. This approximate model replicates the structure of the adaptive path from the primary source,  $ref(k)$ , to the output of the secondary path,  $y(k)$ . Note that for a given disturbance input, there is an “optimal” (but unknown) setting of adaptive filter parameters for which the difference between the primary path and its approximate model is minimized. Finding this *optimal setting* is the objective of the estimation based approach which can be summarized as follows:

1. Devise an estimation strategy that recursively improves the estimate of the optimal values of the adaptive filter parameters in the approximate model of the primary path,
2. Set the actual value of the weight vector in the adaptive filter to the best available estimate of the parameters obtained from the estimation strategy.

Note that in Figure 1,

$$\mathbf{e}(k) = \mathbf{d}(k) - \mathbf{y}(k) + \mathbf{v}_m(k) \quad (2)$$

where (a)  $\mathbf{e}(k) \in \mathcal{R}^{M \times 1}$  is the measured error vector, (b)  $\mathbf{v}_m(k) \in \mathcal{R}^{M \times 1}$  is the exogenous disturbance that captures measurement noise, modeling error and

uncertainty in the initial condition of the secondary path, and (c)  $\mathbf{y}(k) = \mathbf{S}(k) \oplus \mathbf{u}(k)$  (also in  $\mathcal{R}^{M \times 1}$ ) is the output of the secondary path.  $\mathbf{S}(k)$  is the impulse response of the secondary path and  $\oplus$  denotes convolution. Here  $\mathbf{u}(k) = \mathcal{X}^*(k)\mathbf{W}(k)$  where  $\mathcal{X}(k) = \text{diag}(\mathbf{X}(k) \cdots \mathbf{X}(k))$  and

$$\mathbf{W}(k) = [W_{11}^T(k) \cdots W_{1J}^T(k) \cdots W_{KJ}^T(k)]^T \quad (3)$$

is the weight vector of the MIMO adaptive filter. Equation (2) can be rewritten as

$$\mathbf{e}(k) + \mathbf{y}(k) = \mathbf{d}(k) + \mathbf{v}_m(k) \quad (4)$$

where the left hand side is a noisy measurement of the output of the primary path  $\mathbf{d}(k)$ . Since  $\mathbf{y}(k)$  is not directly measurable (neither is  $\mathbf{d}(k)$ ), the adaptive algorithm should generate an internal copy of  $\mathbf{y}(k)$  (referred to as  $\mathbf{y}_{copy}(k)$ ). The *derived* measured quantity can then be defined as

$$\mathbf{m}(k) \triangleq \mathbf{e}(k) + \mathbf{y}(k) = \mathbf{d}(k) + \mathbf{v}_m(k) \quad (5)$$

which will be used in formulating the estimation problem. The only assumption involved in constructing  $\mathbf{m}(k)$  is the assumed knowledge of the initial condition of the secondary path. In [7] it is shown that for a linear, stable secondary path (a realistic assumption in practice), any error in  $\mathbf{y}(k)$  due to an initial condition different from what is assumed by the algorithm remains bounded (hence it can be treated as a component of the measurement disturbance). Furthermore, for sufficiently large  $k$ , this error decays to zero (i.e.  $\mathbf{y}_{copy}(k) \rightarrow \mathbf{y}(k)$ ). The state space model for the secondary path is

$$\theta(k+1) = A_s(k)\theta(k) + B_s(k)\mathbf{u}(k) \quad (6)$$

$$\mathbf{y}(k) = C_s(k)\theta(k) + D_s(k)\mathbf{u}(k) \quad (7)$$

where  $\theta(k)$  is the state variable capturing the dynamics of the secondary path. The weight vector of the adaptive filter  $\mathbf{W}(k)$  is also treated as the state vector that captures the dynamics of the FIR filter. Note that  $W_{kj}(k)$  is itself a vector of length  $L$  (length of each FIR filter).  $\xi_k^T = [\mathbf{W}^T(k) \ \theta^T(k)]$  is then the

state vector for the overall system. The state space representation of the system is then

$$\begin{bmatrix} \mathbf{W}(k+1) \\ \theta(k+1) \end{bmatrix} = \begin{bmatrix} I_{(JKL) \times (JKL)} & 0 \\ B_s(k)\mathcal{X}^*(k) & A_s(k) \end{bmatrix} \begin{bmatrix} \mathbf{W}(k) \\ \theta(k) \end{bmatrix} \triangleq F_k \xi_k \quad (8)$$

where  $\mathcal{X}(k)$ , captures the effect of the reference input vector  $\mathbf{X}(k)$ . For this system, the *derived* measured output is

$$\begin{aligned} \mathbf{m}(k) &= [D_s(k)\mathcal{X}^*(k) \ C_s(k)] \begin{bmatrix} \mathbf{W}(k) \\ \theta(k) \end{bmatrix} + \mathbf{v}_m(k) \\ &\triangleq H_k \xi_k + \mathbf{v}_m(k) \end{aligned} \quad (9)$$

where  $\mathbf{m}(k)$  is defined in Equation (5). Noting the objective of the adaptive filtering problem in Fig. 1,  $\mathbf{s}(k) = \mathbf{d}(k)$  is the quantity to be estimated,

$$\begin{aligned} \mathbf{s}(k) &= [D_s(k)\mathcal{X}^*(k) \ C_s(k)] \begin{bmatrix} \mathbf{W}(k) \\ \theta(k) \end{bmatrix} \\ &\triangleq L_k \xi_k \end{aligned} \quad (10)$$

Here  $\mathbf{m}(k) \in \mathcal{R}^{M \times 1}$ ,  $\mathbf{s}(k) \in \mathcal{R}^{M \times 1}$ ,  $\theta(k) \in \mathcal{R}^{N_s \times 1}$  where  $N_s$  is the order of the secondary path. All matrices are then of appropriate dimensions. Equations (8) through (10) are identical to the equations in the single channel case and the only difference is in the dimension of the variable involved [7].

Defining  $\hat{\mathbf{s}}(k|k) \triangleq \mathcal{F}(\mathbf{m}(0), \dots, \mathbf{m}(k))$  as the filtering estimate of  $\mathbf{s}(k)$ , the objective in the filtering solution is to find  $\hat{\mathbf{s}}(k|k)$  such that the worst case energy gain from the measurement disturbance and the initial condition uncertainty to the error in the filtering estimate is properly bounded, i.e.

$$\sup_{\mathbf{v}_m, \xi_0} \frac{\sum_{n=0}^M [\mathbf{s}(k) - \hat{\mathbf{s}}(k|k)]^* [\mathbf{s}(k) - \hat{\mathbf{s}}(k|k)]}{(\xi_0 - \hat{\xi}_0)^* \Pi_0^{-1} (\xi_0 - \hat{\xi}_0) + \sum_{n=0}^M \mathbf{v}_m^*(k) \mathbf{v}_m(k)} \leq \gamma_f^2 \quad (11)$$

In a similar way, defining  $\hat{\mathbf{s}}(k) \triangleq \mathcal{F}(\mathbf{m}(0), \dots, \mathbf{m}(k-1))$  as the prediction estimate of  $\mathbf{s}(k)$ , the objective in the prediction solution is to find  $\hat{\mathbf{s}}(k)$  such that

$$\sup_{\mathbf{v}_m, \xi_0} \frac{\sum_{n=0}^M [\mathbf{s}(k) - \hat{\mathbf{s}}(k)]^* [\mathbf{s}(k) - \hat{\mathbf{s}}(k)]}{(\xi_0 - \hat{\xi}_0)^* \Pi_0^{-1} (\xi_0 - \hat{\xi}_0) + \sum_{n=0}^M \mathbf{v}_m^*(k) \mathbf{v}_m(k)} \leq \gamma_p^2 \quad (12)$$

Solutions to these two problems are discussed in detail in Reference [7], and therefore the next section only briefly presents the simplified solutions.

### 3 $H_\infty$ -Optimal Solution

**Theorem 1:** For the state space representation described by Eqs. (8)-(10), and for  $L_k = H_k$ , the central  $H_\infty$ -optimal solution to the filtering problem in Eq. (11) is obtained for  $\gamma_f = 1$ , and is given by

$$\hat{\xi}_{k+1} = F_k \hat{\xi}_k + K_{f,k} (m(k) - H_k \hat{\xi}_k), \quad \hat{\xi}_0 = 0 \quad (13)$$

$$\hat{s}(k|k) = L_k \hat{\xi}_k + (L_k P_k H_k^*) R_{He,k}^{-1} (m(k) - H_k \hat{\xi}_k) \quad (14)$$

$$K_{f,k} = (F_k P_k H_k^*) R_{He,k}^{-1} \text{ and } R_{He,k} = I_p + H_k P_k H_k^* \quad (15)$$

where  $P_k$  satisfies the Lyapunov recursion  $P_{k+1} = F_k P_k F_k^* + \Pi_0$ .

**Theorem 2:** For the state space representation described by Eqs. (8)-(10), and for  $L_k = H_k$ , if  $(I - P_k L_k^* L_k) > 0$ , then the central  $H_\infty$ -optimal prediction solution to the linearized problem is obtained for  $\gamma_p = 1$ , and is given by

$$\hat{\xi}_{k+1} = F_k \hat{\xi}_k + K_{p,k} (m(k) - H_k \hat{\xi}_k), \quad \hat{\xi}_0 = 0 \quad (16)$$

$$\hat{s}(k) = L_k \hat{\xi}_k \quad (17)$$

with  $K_{p,k} = F_k P_k H_k^*$  where  $P_k$  satisfies the Lyapunov recursion  $P_{k+1} = F_k P_k F_k^* + \Pi_0$ .

### 4 Implementation Outline

The implementation algorithm is identical to that in the single channel case [7].

1. Start with  $\hat{W}(0) = 0$  and  $\hat{\theta}(0) = 0$  (the initial guess for the state vector in the approximate model of the primary path),  $\theta_{copy}(0) = 0$ , and  $h(0) = [x(0) \ 0 \ \dots \ 0]^T$ . The initial value for the Riccati matrix is  $P_0$  which is chosen to be block diagonal. The role of  $P_0$  is similar to the learning rate in LMS-based adaptive algorithms [7].
2. If  $0 \leq k \leq M$  (finite horizon):

- (a) Form the control signal

$$u(k) = \mathcal{X}^*(k) \hat{W}(k) \quad (18)$$

to be applied to the secondary path to produce

$$y(k) = C_s(k) \theta(k) + D_s(k) u(k) \quad (19)$$

at the output of the secondary path. The error signal (available for the state update at time  $k$ ) is then:

$$e(k) = d(k) - y(k) + v_m(k) \quad (20)$$

- (b) Propagate the state estimate and the internal copy of the state of the secondary path as follows

$$\begin{bmatrix} \hat{W}(k+1) \\ \hat{\theta}(k+1) \\ \theta_{copy}(k+1) \end{bmatrix} = \begin{bmatrix} F_k + K_{f,k} [0 \ -C_s(k)] & K_{f,k} C_s(k) \\ [B_s(k) \mathcal{X}^*(k) \ 0] & A_s(k) \end{bmatrix} \times \begin{bmatrix} \hat{W}(k) \\ \hat{\theta}(k) \\ \theta_{copy}(k) \end{bmatrix} + \begin{bmatrix} K_{f,k} \\ 0 \end{bmatrix} e(k) \quad (21)$$

where  $e(k)$  is the error sensor measurement at time  $k$  given by Eq. (20), and  $K_{f,k} = F_k P_k H_k^* (I + H_k P_k H_k^*)^{-1}$  (see Theorem 1). Note that for the prediction-based EBAF algorithm we only need to replace  $K_{f,k}$  with  $K_{p,k} = F_k P_k H_k^*$ .

- (c) Update the Riccati matrix  $P_k$  using the Lyapunov recursion.  $P_{k+1}$  will be used in (21) to update the state estimate.

3. Go to (2)

### 5 Simulation Results

In this section the performance of the proposed multi-channel EBAF algorithm in two applications is investigated. Comparisons are made to the multi-channel implementation of the FxLMS adaptive algorithm in each application.

#### 5.1 Active Vibration Isolation

The Vibration Isolation Platform (VIP) is an experimental set up which is designed to capture the main features of real world payload isolation and pointing problem. Payload isolation refers to the vibration isolation of payload structures with instruments or equipments requiring a very quiet mounting [3]. VIP is designed such that the base supporting the payload can emulate spacecraft dynamics. Broadband as well as narrowband disturbances can be introduced to the middle mass (emulating real world vibration sources such as solar array drive assemblies, reaction wheels, etc.) via a set of three voice coil actuators. The positioning of a second set of voice coil actuators allows

for the implementation of an adaptive/active isolation system. For more description of VIP see [7].

In all simulations that follow, the length of the adaptive FIR filters, is 4. This length is found to be sufficient for an acceptable performance of the adaptive algorithms. The sampling frequency for all the simulations in this section is 1000 Hz. Furthermore, all measurements are subject to band limited white noise with power 0.008.

Figures 2 and 3 show the reading of the scoring sensors when the primary disturbances are multi-tone sinusoids. The primary disturbance consists of sinusoidal signal of amplitudes 0.1 and 0.2 volts at 4 and 15 Hz, respectively. Both components of excitation for the disturbance actuator #1 are assumed to have zero phase. Each sinusoidal component of the excitation for the disturbance actuator #2 (#3) is assumed to have a phase lag of 22.5 (45) degrees with respect to the corresponding component of the excitation in actuator #1. Figures 2 and 3 demonstrate a trend similar to the single tone scenario. For the FxLMS algorithm, a trade off between the amplitude of the transient vibrations of the payload and the speed of the convergence exists. The adaptation rate here is 0.0001. Slower adaptation rates can reduce the amplitude of the transient vibrations at the expense of the speed of the convergence. For the EBAF algorithm, however, better transient behavior and faster convergence are achieved without compromising the steady state performance. In the steady state, the EBAF algorithm provides a 15 times reduction in the amplitude of the vibrations of the payload. For the FxLMS algorithm a 9 times reduction is recorded.

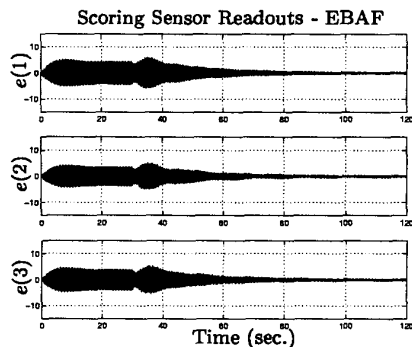
## 5.2 Active Noise Cancellation

The objective of the multi-channel noise cancellation, see Figure 4, is to use both available speakers to simultaneously cancel the effect of the incoming disturbance (entering via speaker #1) at Microphones #1 and #2. The control signal is supplied to each speaker via an adaptive FIR filter (*i.e.* in

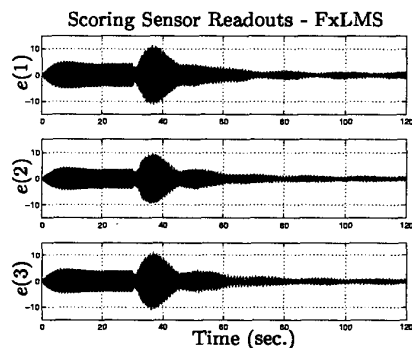
the case of Speaker #1 added to the primary disturbance). Figure 5 shows the output of the microphones when the primary disturbance (applied to Speaker #1) is a multi-tone sinusoid with 150 Hz and 200 Hz frequencies. The length of each FIR filter in this simulation is 8. For the first two seconds the controller is off (*i.e.* both adaptive filters have zero outputs). At  $t = 2.0$  seconds the controller is switched on. The initial value for the Riccati matrix is  $P_0 = \text{diag}(0.0005I_{2(N+1)}, 0.00005I_{N_s})$  (where  $N + 1$  is the length of each FIR filter, and  $N_s$  is the order of the secondary path). It is clear that the error at Microphone #2 is effectively canceled in 0.2 seconds. For Microphone #1 however the cancellation time is approximately 5 seconds. A 30 times reduction in disturbance amplitude is measured at Microphone #2 in approximately 10 seconds. For Microphone #1 this reduction is approximately 15 times. Note that the distance between Speaker #2 and Microphone #1 (46 inches) is much greater than the distance between Speaker #1 and Microphone #1 (6 inches). Due to this physical constraint, Speaker #2 alone, is not enough for an acceptable noise cancellation at both microphones. The experimental data in [7] confirm this observation. Using a multi-channel approach however, allows for a substantial reduction in the amplitude of the measured noise at both microphones. Nevertheless, noise cancellation at the position of Microphone #1 tends to be slower than the noise cancellation at the position of Microphone #2.

## 6 Conclusion

A new estimation-based adaptive filtering/control algorithm for the multi-channel Filtered-LMS type problems is presented. The synthesis and analysis of the multi-channel adaptive (FIR) filter are shown to be identical to the single-channel case. Simulations for a 3-input/3-output Vibration Isolation Platform (VIP), and a multi-channel noise cancellation in a one dimensional acoustic duct are used to demonstrate the feasibility of the estimation-based approach. The new



**Fig. 2:** Performance of a multi-channel implementation of EBAF algorithm (disturbance at 4 and 15 Hz). The reference signal is contaminated with band limited white noise (SNR=4.5). The control signal is applied for  $t \geq 30$  seconds.

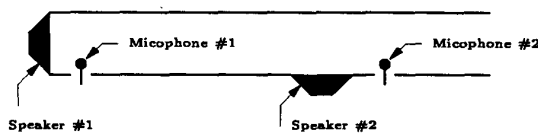


**Fig. 3:** Simulation scenario in Figure 2 under MIMO implementation of FxLMS.

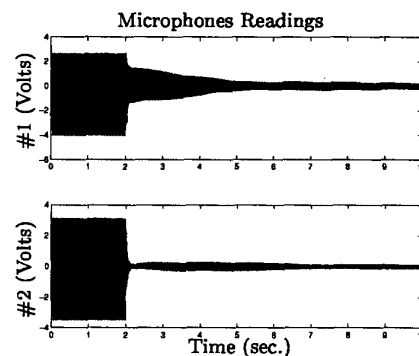
estimation-based adaptive filtering algorithm is shown to provide both faster convergence (with acceptable transient behavior), and improved steady state performance when compared to a multi-channel implementation of the FxLMS algorithm.

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**Fig. 4:** Schematic diagram of one-dimensional air duct



**Fig. 5:** Performance of the Multi-Channel noise cancellation in acoustic duct for a multi-tone primary disturbance at 150 and 200 Hz. The control signal is applied for  $t \geq 2$  seconds.

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