

Modified Fincke-Pohst Algorithm for Low-Complexity Iterative Decoding over Multiple Antenna Channels

Haris Vikalo
Information Systems Lab
Stanford University
Stanford, CA 94305, USA
e-mail: hvikalo@stanford.edu

Babak Hassibi
Department of Electrical Engineering
California Institute of Technology
Pasadena, CA 91125, USA
e-mail: hassibi@caltech.edu

Abstract — In recent years, soft iterative decoding techniques have been shown to greatly improve the bit error rate performance of various communication systems. For multiple antenna systems employing space-time codes, however, it is not clear what is the best way to obtain the soft-information required of the iterative scheme with low complexity. In this paper, we propose a modification of the Fincke-Pohst (sphere decoding) algorithm [1] to estimate the maximum a posteriori (MAP) probability of the received symbol sequence. The new algorithm (FP-MAP) solves a nonlinear integer least-squares problem and, over a wide range of rates and SNRs, has polynomial-time (often cubic) expected complexity. The FP-MAP algorithm provides soft detection information for the soft channel decoder. The soft decoder's output is then fed back to the FP-MAP, and iterated on. The performance of the FP-MAP algorithm on a multiple antenna system employing turbo code is demonstrated.

I. INTRODUCTION

We assume a discrete-time block fading multiple antenna channel model with M transmit and N receive antennas, where the transmitted signal \mathbf{s} and received signal \mathbf{x} are related by

$$\mathbf{x} = \mathbf{H}\mathbf{s} + \mathbf{v},$$

where \mathbf{H} is the channel matrix, and \mathbf{v} is the additive noise vector. Furthermore, transmitted symbols \mathbf{s} are obtained upon modulating a coded sequence $\{c_i\}$ onto an m -dimensional ($m = 2M$) subset of a lattice \mathcal{D}_L^m (lattice subset \mathcal{D}_L^m spans over L points in each of m dimensions).

For a known channel in AWGN, MAP detection is equivalent to the optimization problem

$$\min_{\mathbf{s} \in \mathcal{D}_L^m} \left[\|\mathbf{x} - \mathbf{H}\mathbf{s}\|^2 - \sum_{k=1}^m \log p(s_k) \right], \quad (1)$$

where $p(s_k)$ denotes a priori probability of a k^{th} entry in a symbol vector. Applying the idea of Fincke-Pohst algorithm (which has polynomial expected complexity [2]), rather than to search over the entire lattice, we search only over lattice points \mathbf{s} that belong to the geometric body described by

$$r^2 \geq (\mathbf{s} - \hat{\mathbf{s}})^* R^* R (\mathbf{s} - \hat{\mathbf{s}}) - \sum_{k=1}^m \log p(s_k), \quad (2)$$

where R is lower triangular matrix following QR factorization of H and $\hat{\mathbf{s}}$ is standard least-squares estimate. [Note that, unlike in the original sphere decoder algorithm, this geometric body is no longer a hypersphere.] The search radius r in (2)

can be chosen according to the statistical properties of the noise and a priori distribution of \mathbf{s} .

For an iterative decoding scheme, we require soft information, i.e., probability that each bit is decoded correctly. To this end, consider log-likelihood ratio of the form

$$L(c_i|\mathbf{x}) = \log \frac{\sum_{\mathbf{s}: c_i = +1} e^{-\|\mathbf{x} - \mathbf{H}\mathbf{s}\|^2 + \sum_j \log p[s_j]}}{\sum_{\mathbf{s}: c_i = -1} e^{-\|\mathbf{x} - \mathbf{H}\mathbf{s}\|^2 + \sum_j \log p[s_j]}} \quad (3)$$

Computing (3) over entire signal space \mathcal{D}_L^m is of prohibitive complexity. Instead, we constrain ourselves to those $\mathbf{s} \in \mathcal{D}_L^m$ for which argument in (1) is small. [Note that these are the signal vectors whose contribution to the numerator and denominator in (3) is significant.]

Assume that the search in (2) yields the set of points $\mathcal{S} = \{s^{(1)}, s^{(2)}, \dots, s^{(l)}\}$. The vector $\mathbf{s} \in \mathcal{S}$ that minimizes (1) is the solution to the MAP detection problem. The soft information for each bit c_i can be estimated from (3), by only summing the terms in the numerator and denominator such that $\mathbf{s} \in \mathcal{S}$.

Figure 1 shows the bit-error performance of the system with 4 transmit and 4 receive antennas, 16-QAM constellation and parallel concatenated turbo code with rate $R = 1/2$ and length 9216 information bits. For each iteration of the FP-MAP, turbo (inner) decoder performs 8 iterations of its own.

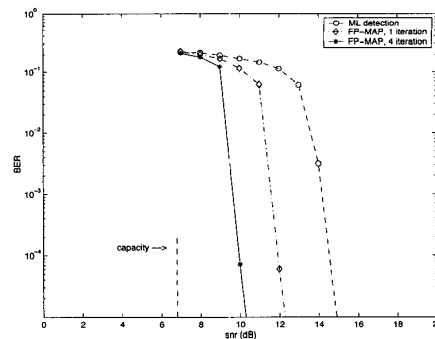


Figure 1: Rate 1/2 turbo code, 4 × 4 system, 16-QAM

ACKNOWLEDGMENTS

Thanks to Hui Jin for useful discussions.

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