INDEFINITE METRIC SPACES IN ESTIMATION, CONTROL AND ADAPTIVE FILTERING

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I certify that I have read this thesis and that in my opinion it is fully adequate, in scope and in quality, as a dissertation for the degree of Doctor of Philosophy.

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Abstract

The goal of this thesis is two-fold: first, to present a unified mathematical framework (based upon optimization in indefinite metric spaces) for a wide range of problems in estimation and control, and second, to motivate and introduce the problem of robust estimation and control, and to study its implications to the area of adaptive signal processing.

Robust estimation (and control) is concerned with the design of estimators (and controllers) that have acceptable performance in the face of model uncertainties and lack of statistical information, and can be considered an outgrowth and extension of (the now classical) LQG theory, developed in the 1950's and 1960's, which assumed perfect models and complete statistical knowledge. It has particular significance in adaptive signal processing where one needs to cope with time-variations of system parameters and to compensate for lack of a priori knowledge of the statistics of the input data and disturbances. One method of addressing the above problem is the so-called H^∞ approach, which was introduced by G. Zames in 1980 and that has been recently solved by various authors.

Despite the "fundamental differences" between the philosophies of the H^∞ and LQG approaches to control and estimation, there are striking "formal similarities" between the controllers and estimators obtained from these two methodologies. In an attempt to explain these similarities, we shall describe a new approach to H^∞ estimation (and control), different from the existing (e.g., interpolation-theoretic-based, game-theoretic-based, etc.) approaches, that is based upon setting up estimation (and control) problems, not in the usual Hilbert space of random variables, but in an indefinite (so-called Krein) space.

The Krein space formulation provides a unified approach for problems in LQG, H^∞, risk-sensitive, and game-theoretic, estimation and control, and, most importantly, allows one to use the insight obtained from over three decades of work in traditional LQG theory to obtain new results in these other areas. Proceeding in this spirit, we demonstrate how to generalize the (possibly) numerically superior square-root algorithms and the fast Chandrasekhar algorithms to the H^∞ setting, and embark on some new investigations on the asymptotic behaviour of H^∞ filters and controllers, and on the existence and properties of solutions of Riccati equations with (possibly) indefinite coefficient matrices.

We also study adaptive filtering using the H^∞ approach to robust estimation and show that the celebrated LMS (least-mean squares) adaptive algorithm is H^∞-optimal. This result solves the long standing issue of finding a rigorous basis for LMS (which was long thought to be an approximate least-squares solution). It also suggests some further ramifications, such as the design of robust adaptive filters with more desirable tracking properties, as well as some directions for further research, such as the mixed H^2/H^∞ problem.

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